Exam. Code : 211004

Subject Code : 4636

M.Sc. (Mathematics) 4th Semester

DISCRETE MATHEMATICS—I

Paper—MATH–575

Time Allowed—2 Hours] [Maximum Marks—100

Note :— Attempt any **four** questions. All questions carry equal marks.

1. (a) Let A be the set of integers and Let R be a relation on A × A defined by (a, b) R(c, d) if a + d = b + c. Prove that R is an equivalence relation.

   (b) Let A = { 1, 2, 3, 4, 5, 6} and Let R be an equivalence relation on A defined by :

   R = { (1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (6, 2), (6, 3), (6, 6)}.

   Find the equivalence classes of R.

2. (a) Let A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be ordered by divisibility. Draw the Hasse diagram of A.

(b) How many integers between 1 and 300 (both 1 and 300 are included) are :

(i) Divisible by at least one of 3, 5, 7 ?

(ii) Divisible by 3 and 5, but not by 7 ?

(iii) Divisible by 5 but neither by 3 nor by 7 ?

3. (a) Using truth tables, determine whether the following statements are tautology, contingency, and a contradiction :

(i) $p \rightarrow (p \rightarrow q)$

(ii) $p \rightarrow (q \rightarrow p)$

(iii) $p \wedge (\sim p)$.

(b) Without using truth tables, prove that the argument :

$p \rightarrow \sim q, r \rightarrow q, r \vdash \sim p$ is valid.

4. (a) Translate the following argument into symbolic form :

"If 6 is even then 2 does not divide 7. Either 5 is not prime or 2 divides 7. But 5 is prime, therefore, 6 is odd."

Using truth table, test the validity of the above argument.

(b) Let $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of each of the following statements :

(i) $(\exists x \in A)\ (x + 3 = 10)$

(ii) $(\forall x \in A)\ (x + 3 < 10)$

(iii) $(\exists x \in A)\ (x + 3 < 5)$

(iv) $(\forall x \in A)\ (x + 3 \leq 7)$

Also write the negation of all these four statements.

5. (a) Consider an algebraic system $< Q, * >$, where Q is the set of rational numbers and * is a binary operation defined by $a * b = a + b - ab$, $\forall a, b \in Q$. Determine whether $< Q, * >$ is a group or not a group.

(b) Let $S = Q \times Q$ be the set of ordered pairs of rational numbers (Q being the set of rational numbers) with the binary operation * defined by :

$$(a, b) * (x, y) = (ax, ay + b)$$

(i) Find $(3, 4) * (1, 2)$ and $(-1, 3) * (5, 2)$.

(ii) Is $< S, * >$ is a semi group ?

(iii) Find the identity element of S.

(iv) Which elements, if any, have inverses and what are they ?

6. (a) Consider G = {1, 3, 7, 9} under multiplication modulo 10 and $f : z_4 \rightarrow G$ be a function defined by :

$f(0) = 1,\ f(1) = 3,\ f(2) = 9,\ f(3) = 7.$

Show that f is a semigroup homomorphism. Also prove that $z_4 \cong G$.

(b) Let z be the set of integers and m > 1 be any positive integer. Consider a relation "Congruence module m" defined by :

$a \equiv b$ modm if $m/(a - b)\ \forall\ a, b \in z$. Show that '≡' is a congruence relation on z.

7. (a) Obtain the recurrence relation of second order from the sequence given by :

$$S_k = 2.4^k - 5.(-3)^k.$$

(b) Solve the recurrence relation :

$a_r - 7\ a_{r-1} + 10\ a_{r-2} = 0$, given that $a_0 = 0$, $a_1 = 3$.

8. (a) Obtain the generating functions for the following sequences $\{S_n\}$, where :

(i) $S_n = 2^n\ ;\ n \geq 0$

(ii) $S_n = n\ ;\ n \geq 0$

(iii) $S_n = ba^n\ ;\ n \geq 0.$

(b) Solve the recurrence relation :

$S(k) - 3\ S(k - 1) - 2 = 0,\ k \geq 1$, where $S(0) = 1$, by using generating function.